

1. The probability of rolling two six-sided dice and having the sum on the two dice equal 7 is $\frac{1}{6}$.

(a) Interpret this probability.

If two dice were rolled repeatedly (really), the proportion of rolls that had a sum of 7 on the two dice would be approximately $\frac{1}{6}$.

(b) You roll two dice six times. Are you guaranteed to get a sum of 7 once? Explain.

no. We can only predict the proportion of 7s that are rolled in the "long run." This short run can not be predicted.

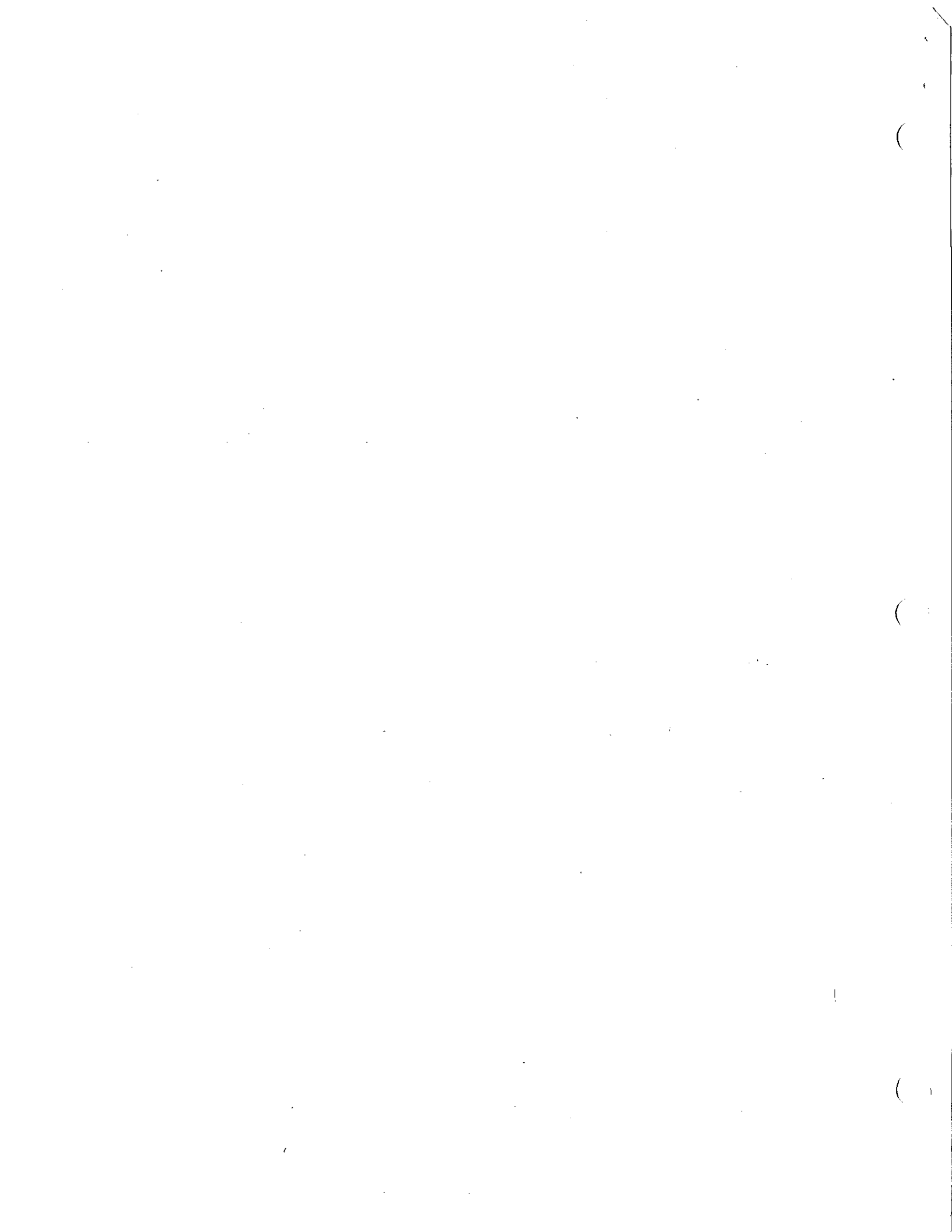
2. To pass the time during a long drive, you and a friend are keeping track of the makes and models of cars that pass by in the other direction. At one point, you realize that among the last 20 cars, there hasn't been a single Ford. (Currently, about 16% of cars sold in America are Fords). Your friend says, "The law of averages says that the next car is almost certain to be a Ford." Explain to your friend what he doesn't understand about probability.

The next car doesn't know it is supposed to be a Ford.

Assuming that makes of cars on this road are independent of other cars, the fact that the last car (or even 20) were not Fords does not change the probability that the next car is a Ford.

3. A bag contains 10 equally-sized tags numbered 0 to 9. You reach in and, without looking, pick 3 tags without replacement. We want to use simulation to estimate the probability that the sum of the 3 numbers is at least 18. Describe the simulation procedure below, then use the random number table on the next page to carry out 10 trials of your simulation and estimate the probability. Mark on or above each line of the table so that someone can clearly follow your method.

The 0 to 9 numbered on the tags correspond to the single digits 0-9 on the table. Select 3 unique digits from the table in a row (ignore repeats). Record the sum of these 3 digits and repeat for a total of 10 trials. Count the number of trials in which the sum was at least 18. Setup a proportion of times that the sum was 18 or greater.



Random number table for question 3.

128	15689	14227	06565	14374	13852	49367	81982	87209
129	36759	58984	68288	22913	18638	54303	00795	08727
130	69051	64817	87174	09517	84534	06489	87201	97245
131	05007	16632	81194	14873	04197	85576	45195	96565

Trial	digits	sum
1	1 5 6	12
2	8 9 1	18 →
3	4 2 7	13
4	0 6 5	11
5	6 5 1	12
6	4 3 7	14
7	4 1 3	8
8	3 5 2	10
9	4 9 3	16 →
10	6 7 8	21 →

$\frac{2}{10}$ are at least 18.

According to our simulation we can estimate that about 20% of the time the sum of the 3 digits is at least 18.

1. The probability of flipping four coins and getting four "heads" is $\frac{1}{16}$.

(a) Interpret this probability.

If 4 coins were flipped repeatedly for a very long time the proportion of flips that had four heads would be about $\frac{1}{16}$.

(b) You flip four coins 32 times. Are you guaranteed to get four "heads" twice? Explain.

No. The proportion of all flip that end with 4 heads can be estimated to be $\frac{1}{16}$ out of 16. In a short trial of 32, we can not predict the outcomes.

2. You are playing a board game with some friends in which each turn begins with rolling two dice. In this game, rolling "doubles"—the same number on both dice—is especially beneficial. You've rolled doubles on your last three turns, and one of your friends says, "No way you'll roll doubles this time, it would be nearly impossible." Explain to your friend what he doesn't seem to understand about probability.

Each roll of the dice is independent. The previous rolls have no influence on the next roll. The dice have no "memory" of the earlier rolls.

We can only make some predictions on the long run of rolling two dice, not individual rolls.

3. A school's debate club has 10 members, 6 females and 4 males. If the team decides to pick two members randomly to participate in a debate, what is the probability that both of the chosen members are female? We want to use simulation to estimate this probability. Describe the simulation procedure below, then use the random number table on the next page to carry out 10 trials of your simulation and estimate the probability. Mark on or above each line of the table so that someone can clearly follow your method.

Randomly Assign the digits 0 through 5 to the club's female members and 6 to 9 to the males. Choose two numbers at a time from the random digits table. Ignore repeats. Record the numbers and the genders they represent. Repeat this process for at least 20 trials. Count how many times both members were female.

Random number table for question 3.

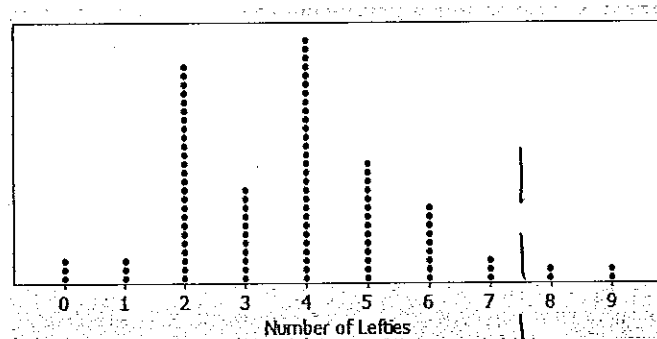
141	96767	35964	23822	96012	(94591	65194	50842	53372
142	72829	50232	97892	63408	77919	44575	24870	04178
143	88565	42628	17797	49376	61762	16953	88604	12724
144	62964	88145	83083	69453	46109	59505	69680	00900

Trial	digits	2 females? ✓
1	96	—
2	76	—
3	73	✗
4	59	—
5	64	—
6	23	✓
7	82	—
8	29	—
9	60	—
10	12	✓

$\frac{2}{10}$

According to the simulation approximately 20% of the time 2 females will be randomly chosen for the debate.

- (c) Below are the number of left-handers in 100 simulated classes of 28 students, assuming that students are selected randomly from a population in which 14% of individuals are left-handed. What do these results suggest about the proportion of lefties in Mr. Millar's class?



1. Suppose you choose a random U.S. resident over the age of 25. The table below is a probability model for the education level the selected person has attained, based on data from the American Community Survey from 2006-2008.

Education level attained	Probability
No high school diploma	0.20
High School diploma or GED	0.22
Some college	0.29
Bachelor's degree	0.19
Graduate or professional degree	? 0.1

- (a) What is the probability that a randomly selected person has a graduate or professional degree? (That is, fill in the space marked with a "?") Show your work.

$$\begin{aligned}
 P(\text{None}) + P(\text{Dip}) + P(\text{Some}) + P(\text{Bach}) + P(\text{graduate}) &= 1 \\
 0.2 + 0.22 + 0.29 + 0.19 + X &= 1 \\
 0.9 + X &= 1 \\
 X &= 0.1
 \end{aligned}$$

- (b) What is the probability that a randomly-selected person has at least a high school diploma? Show your work.

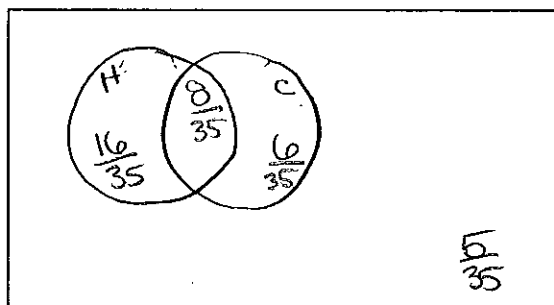
$$\begin{aligned}
 P(\text{at least HS}) &= P(\text{HS}) + P(\text{Some college}) + P(\text{Bach}) + P(\text{Graduate}) \\
 &= 0.22 + 0.29 + 0.19 + 0.1 \\
 &= 0.8
 \end{aligned}$$

or $P(\text{at least HS}) = 1 - P(\text{no high school})$

2. There are 35 students in Ms. Ortiz's Precalculus class. One day, 24 students turned in their homework and 14 turned in test corrections. Eight of these students turned in both homework and test corrections. Suppose we randomly select a student from the class.

- (a) Fill in the Venn diagram below so that it describes the chance process involved here. Let H = the event "turned in homework" and C = the event "turned in corrections."

16 + 8 + 6 = X
Some must have turned in nothing in



$n = 35$

- (b) What is the probability that a randomly-chosen student turned in neither homework nor corrections? Justify your answer with appropriate calculations.

$$\begin{aligned}
 P(\text{neither}) &= 1 - [P(H) + P(C) + P(\text{both})] \\
 &= 1 - \left[\frac{16}{35} + \frac{8}{35} + \frac{6}{35} \right] = \frac{1}{7} \\
 &= 1 - \left[\frac{30}{35} \right] \\
 &= \frac{5}{35}
 \end{aligned}$$

3. Below is a two-way table that describes responses of 120 subjects to a survey in which they were asked, "Do you exercise for at least 30 minutes four or more times per week?" and "What kind of vehicle do you drive?"

		Car type			Total
		Sedan	SUV	Truck	
Exercise?	Yes	25	15	12	52
	No	20	24	24	68
Total		45	39	36	120

Suppose one person from this sample is randomly selected.

- (a) List two mutually exclusive events for this chance process.

Sedan and SUV

... others.

- (b) What is the probability that the person selected drives an SUV?

$$P(\text{SUV}) = \frac{39}{120} = \frac{13}{40}$$

- (c) What is the probability that the person selected drives either a sedan or a truck?

$$P(\text{Sedan or truck}) = P(\text{Sedan}) + P(\text{truck}) = \frac{45}{120} + \frac{36}{120} = \frac{81}{120} = \frac{27}{40}$$

mutually exclusive

- (d) What is the probability that the person selected drives a truck or exercises four or more times per week?

$$P(\text{truck or yes}) = P(\text{truck}) + P(\text{yes}) - P(\text{truck \& yes})$$

not mutually exclusive.

$$= \frac{36}{120} + \frac{52}{120} - \frac{12}{120} = \frac{76}{120} = \frac{19}{30}$$

1. The table below is a probability model for the number of cars in a randomly-selected household in the United States., (Based on U.S. Census 2000 data).

Number of cars	0	1	2	3	4	5 or more
Probability	0.07	0.19	0.47	?0.19	0.06	0.02

(a) What is the probability that a randomly selected household has three cars? (That is, fill in the space marked with a "?") Show your work.

$$\begin{aligned}
 P(0) + P(1) + P(2) + P(3) + P(4) + P(5 \text{ or more}) &= 1 \\
 0.07 + 0.19 + 0.47 + x + 0.06 + 0.02 &= 1 \\
 x + 0.81 &= 1 \\
 x &= 0.19
 \end{aligned}$$

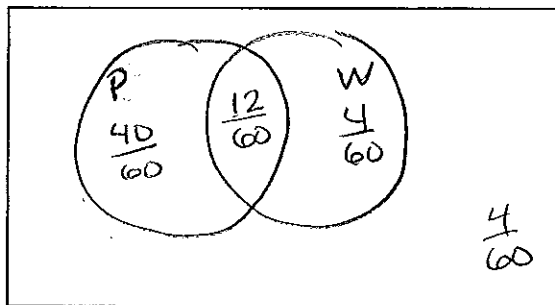
(b) What is the probability that a randomly-selected household has at least 2 cars? Show your work.

$$\begin{aligned}
 P(\text{at least 2 cars}) &= P(2) + P(3) + P(4) + P(5 \text{ or more}) \\
 &= 0.47 + 0.19 + 0.06 + 0.02 \\
 &= 0.74
 \end{aligned}$$

or $P(\text{at least 2}) = 1 - P(\text{less than 2})$

2. Last Saturday at Pasquale's Pizzas and Wings, 60 customers were served over the course of the evening. Fifty-two customers ordered pizza and 16 ordered buffalo wings. Twelve of these customers ordered both pizza and wings. Suppose we select a customer from last Saturday at random.

(a) Fill in the Venn diagram below so that it describes the chance process involved here. Let P = the event "ordered pizza" and W = the event "ordered wings."



- (b) What is the probability that a randomly-chosen customer did not order wings or pizza? Justify your answer with appropriate calculations.

$$\begin{aligned}
 P(\text{not pizza or wings}) &= 1 - P(P \cup W) \\
 &= 1 - \left[\frac{56}{60} \right] \\
 &= \frac{4}{60} = \frac{1}{15} \approx 0.067
 \end{aligned}$$

look at diagram

3. The table below gives the counts (in thousands) of earned degrees in the United States in a recent year, classified by level and by the gender of the degree recipient.

	Degree				Total
	Bachelor's	Master's	Professional	Doctoral	
Female	616	194	30	16	856
Male	529	171	44	26	770
Total	1145	365	74	42	1626

Suppose one degree recipient from this group is selected randomly.

- (a) List two mutually exclusive events for this chance process.

Female
Male

- (b) What is the probability that the person selected earned a Master's degree?

$$P(\text{Masters}) = \frac{365}{1626}$$

- (c) What is the probability that the person selected earned a Professional or Doctoral degree?

$$\begin{aligned}
 P(\text{Pro or Doc}) &= P(\text{Pro}) + P(\text{Doc}) \\
 &= \frac{74}{1626} + \frac{42}{1626} = \frac{116}{1626} = \frac{58}{813}
 \end{aligned}$$

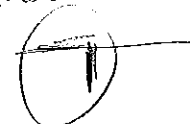
mutually exclusive

- (d) What is the probability that the person selected is female or earned a Master's degree?

not mutually exclusive

$$\begin{aligned}
 P(\text{Fem or Masters}) &= P(F) + P(\text{Masters}) - P(\text{both}) \\
 &= \frac{856}{1626} + \frac{365}{1626} - \frac{194}{1626} \\
 &= \frac{1027}{1626}
 \end{aligned}$$

OR Read Chart



1. Suppose you toss one coin and roll one six-sided die.

(a) List the outcomes in the sample space.

H1 H4 T1 T4
 H2 H5 T2 T5
 H3 H6 T3 T6

(b) Find the probability of getting a head.

12 outcomes

6 have heads $P(H) = \frac{6}{12} = \frac{1}{2}$

(c) Find the probability of getting a 1, 2, or 3 on the die.

$P(1 \text{ or } 2 \text{ or } 3) = P(1) + P(2) + P(3)$ mutually exclusive
 $P(1 \cup 2 \cup 3) = \frac{2}{12} + \frac{2}{12} + \frac{2}{12} = \frac{6}{12} = \frac{1}{2}$

(d) Find the probability of getting a head or a five.

not mutually exclusive

$P(H \text{ or } 5) = P(H) + P(5) - P(H5)$
 $= \frac{6}{12} + \frac{2}{12} - \frac{1}{12}$
 $= \frac{7}{12}$

2. In a statistics class there are 18 juniors and 10 seniors; 6 of the seniors are females, and 12 of the juniors are males. If a student is selected at random, find the probability of selecting

(a) a junior or a female not mutually exclusive table

$P(Jr \text{ or } F) = P(Jr) + P(F) - P(Jr \cap F)$
 $= \frac{18}{28} + \frac{12}{28} - \frac{6}{28}$
 $= \frac{24}{28} = \frac{6}{7}$

	Females	Males	total
Seniors	6	4	10
Juniors	6	12	18
Total	12	16	28

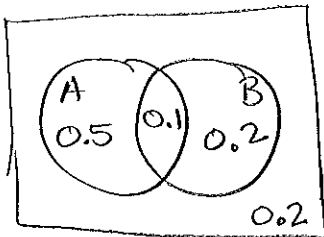
Recall $\frac{6+6+12}{28} = \frac{24}{28} = \frac{6}{7}$

(b) not a junior male

$P(\text{not a junior male}) = 1 - P(\text{Jr and male})$
 $= 1 - \frac{12}{28}$
 $= \frac{16}{28}$
 $= \frac{4}{7}$

3. Consolidated Builders has bid on two large construction contracts. The company president believes that the probability of winning the first contract (event A) is 0.6, that the probability of winning a second (event B) is 0.3, and that the probability of winning both jobs is 0.1.

(a) Construct either a Venn diagram or a two-way table that summarizes what you know about events A and B.



		Event B	
		Y	N
Event A	Y	0.1	0.5
	N	0.2	0.2

(b) What is $P(A \text{ or } B)$ —the probability that Consolidated wins at least one of the job?
both

$$P(A \cup B) = 0.5 + 0.1 + 0.2 = 0.8$$

$$\begin{aligned} \text{or } P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.6 + 0.3 - 0.1 \\ &= 0.8 \end{aligned}$$

(c) Write each of the following events in terms of A, B, A^c , and B^c , and use the information above to calculate the probability of each.

i. Consolidated wins both jobs.

$$P(A \cap B) = 0.1$$

ii. Consolidated wins the first job but not the second.

$$P(A \cap B^c) = 0.5$$

iii. Either Consolidated does not win the first job or wins the second.

$$P(A^c \cup B) = 0.1 + 0.2 + 0.2 = 0.5$$

iv. Consolidated does not win either job.

$$P(A^c \cap B^c) = 0.2$$

1. Ivy conducted a taste test for four different brands of chocolate chip cookies. Below is a two-way table that describes which cookie each subject preferred and their gender.

	Cookie Brand				
	A	B	C	D	Totals
Female	4	6	13	13	36
Male	22	11	11	14	58
Totals	26	17	24	27	94

Suppose one subject from this experiment is selected at random.

- (a) Find the probability that the selected subject preferred Brand C.

$$P(C) = \frac{24}{94} = \frac{12}{47}$$

$$\approx 0.255$$

- (b) Find the probability that the selected subject preferred Brand C, given that she is female.

$$P(C | \text{female}) = \frac{13}{36} \approx 0.361$$

13 = C and female
36 = females

- (c) Are the events "preferred Brand C" and "female" independent? Explain.

By definition if $P(\text{preferred Brand C given female}) = P(\text{preferred Brand C})$ we can say they are independent, but $P(C | \text{female}) \neq P(C)$.
They are not independent.

- (d) Are the events "preferred Brand C" and "female" mutually exclusive? Explain.

No, you can certainly be female and prefer brand C.

$$P(C \cap F) \neq 0$$

- (e) If a random sample of two subjects is selected, what is the probability that neither preferred Brand A?

$$P(A^c \cap A^c) = \frac{68}{94} \cdot \frac{67}{93} = \frac{2278}{4371} \approx 0.521$$

68 people did not choose brand A
67 sample who replaced

2. Officials at Dipstick College are interested in the relationship between participation in (interscholastic) sports and graduation rate. The following table summarizes the probabilities of several events when a male Dipstick student is randomly selected.

Event	Probability
Student participates in sports	0.20
Student participates in sports and graduates	0.18
Student graduates, given no participation in sports	0.82

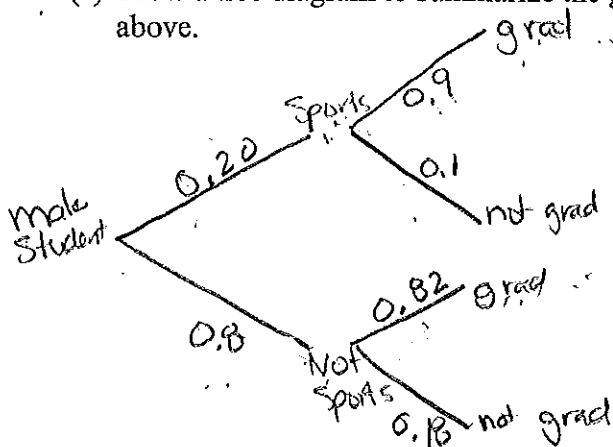
- (a) Find the probability that a student graduates, given that he participates in sports.

$$P(\text{grad}/\text{sports}) = \frac{P(\text{sports and grad})}{P(\text{sports})} = \frac{0.18}{0.20} = \frac{9}{10} = 0.9$$

- (b) Find the probability that the individual does not graduate, given that he participates in sports.

$$P(\text{not grad}/\text{sports}) = 1 - P(\text{grad}/\text{sports}) = 1 - 0.9 = 0.10$$

- (c) Draw a tree diagram to summarize the given probabilities and those you determined above.



- (d) Find the probability that the individual does not participate in sports, given that he graduates.

$$P(\text{not sports}/\text{grad}) = \frac{P(\text{not sports and grad})}{P(\text{grad})} = \frac{(0.8)(0.82)}{(0.2)(0.9) + (0.8)(0.82)} = 0.785$$

1. What age groups use social networking sites? A recent study produced the following data about 768 individuals who were asked their age and which of three social networking sites they used most often. (People who did not use such sites were excluded from the study).

Web site	Age Group (Years)				Totals
	0 – 24	25 – 44	45 – 64	Over 65	
Facebook	77	105	114	12	308
Twitter	46	110	81	7	244
LinkedIn	15	97	95	9	216
Totals	138	312	290	28	768

Suppose one subject from this study was selected at random.

- (a) Find the probability that the selected subject preferred Twitter.

$$P(\text{twitter}) = \frac{244}{768} \approx 0.318$$

- (b) Find the probability that the selected subject preferred Twitter, given that he or she was in the 45 – 64 age group.

$$P(\text{twitter} | 45-64 \text{ age}) = \frac{81}{290} \approx 0.279$$

- (c) Are the events “preferred Twitter” and “age group 45 – 64” independent? Explain.

$$P(\text{twitter} | 45-64 \text{ age}) \neq P(\text{twitter})$$

they are not independent.

- (d) Are the events “preferred Twitter” and “age group 45 – 64” mutually exclusive? Explain.

They are not mutually exclusive, you can be 45-64 and prefer twitter (81 people do).

$$P(\text{Twitter} \cap 45-64) \neq 0$$

- (e) If a random sample of two subjects were selected, what is the probability that neither preferred Twitter?

$$P(T^c \cap T^c) = \frac{524}{760} \cdot \frac{523}{761} \approx 0.465$$

↑ not prefer Twitter
→ no replacement

2. Some days, Ramon drives to work. The rest of the time he rides his bike. Suppose we choose a random work day. The following table gives the probabilities of several events.

Event	Probability
Drives to work	0.20
Drives and is late for work	0.05
Late for work, given he bikes	0.30

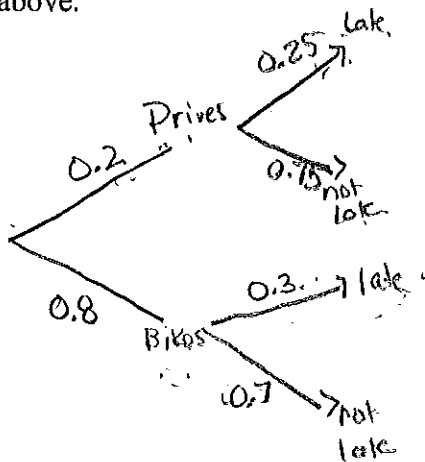
- (a) Find the probability that Ramon is late for work, given that he drives.

$$P(\text{Late} / \text{drives}) = \frac{P(\text{drives and late})}{P(\text{drives})} = \frac{0.05}{0.2} = 0.25$$

- (b) Find the probability that Ramon is not late for work, given that he drives.

$$P(\text{not late} / \text{drives}) = 1 - P(\text{late} / \text{drives}) = 0.75$$

- (c) Draw a tree diagram to summarize the given probabilities and those you determined above.



(d) Find the probability that Ramon drove to work, given that he is late.

$$\begin{aligned} P(\text{Drove} / \text{late}) &= \frac{P(\text{Drove and Late})}{P(\text{late})} = \frac{(0.2)(0.25)}{(0.2)(0.25) + (0.8)(0.3)} \\ &= \frac{0.05}{0.29} \\ &\approx 0.172 \end{aligned}$$

1. Consider the following activity: The letters in the word AARDVARK are printed on identical plastic cards with one letter per card. The eight cards are then placed in a hat, and one card is randomly chosen (without looking) from the hat. The chance process we are interested in is what letter is on the selected card.

(a) List the sample space S of all possible outcomes.

$$S = \{A, A, R, D, V, A, R, K\}$$

(b) Make a table that shows the set of outcomes and the probability of each outcome:

Outcome	A	R	D	V	K
Probability	$\frac{3}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

(c) Consider the following events:

V: the letter chosen is a vowel.

F: the letter chosen falls in the first half of the alphabet (that is, between A and M).

List the outcomes in each of the following events, and determine their probabilities:

$$V = \{A, A, V, A\}$$

$$P(V) = \frac{3}{8}$$

$$F = \{A, D, K, A\}$$

$$P(F) = \frac{5}{8}$$

$$V \text{ or } F = \{A, D, K, A, V, A\}$$

$$P(V \text{ or } F) = \frac{5}{8}$$

$$F^c = \{R, V, R, K\}$$

$$P(F^c) = \frac{3}{8}$$

$$V \text{ and } F = \{A, A\}$$

$$P(V \text{ and } F) = \frac{3}{8}$$

$$V \text{ given } F = \{A, A\}$$

$$P(V|F) = \frac{3}{5}$$

A D K S

(d) Are the events V and F independent? Explain.

$P(V|F) \neq P(V)$ So they are not independent

(e) Are the events V and F mutually exclusive? Explain.

NO, there is a vowel from the first half of the alphabet, A. $P(V \text{ and } F) = \frac{3}{8} \neq 0$

2. Suppose a person was having two surgeries performed at the same time by different operating teams. Assume (unrealistically) that the two operations are independent. If the chances of success for surgery A are 85%, and the chances of success for surgery B are 90%, what is the probability that both will fail?

$$P(A^c \cap B^c) = (0.15)(0.10) = 0.015$$

$$P(A^c) = 1 - P(A) = 0.15$$

$$P(B^c) = 1 - P(B) = 0.10$$

3. Parking for students at Central High School is very limited, and those who arrive late have to park illegally and take their chances at getting a ticket. Joey has determined that the probability that he has to park illegally and that he gets a parking ticket is 0.07. He recorded data last year and found that because of his perpetual tardiness, the probability that he will have to park illegally is 0.25. Suppose that Joey arrived late once again this morning and had to park in a no-parking zone. Can you find the probability that Joey will get a parking ticket? If so, do it. If not, explain what additional information is needed in order to find the probability.

$$P(\text{illegal and ticket}) = 0.07$$

$$P(\text{illegal}) = 0.25$$

$$P(\text{ticket} / \text{illegal}) = \frac{P(\text{illegal and ticket})}{P(\text{illegal})}$$

$$= \frac{0.07}{0.25}$$

$$= 0.28$$